

## 7-6 Rational Zero Theorem

Solve, given that  $-4$  is a zero.  
 $x^3 - 11x + 20 = 0$

$$\begin{array}{r}
 -4 \overline{) 1 \ 0 \ -11 \ 20} \\
 \underline{-4 \ 16 \ -20} \\
 1 \ -4 \ 5 \ 0 \\
 x^2 - 4x + 5 \\
 \frac{+4 \pm \sqrt{16 - 4(1)(5)}}{2} \\
 \frac{4 \pm 2i}{2} \quad \{2 \pm i, -4\}
 \end{array}$$

Warm Up

1. Sketch a graph of a polynomial function of degree 6, max # of turning points, with 4 real zeros and a negative leading coefficient.

2. Sketch a graph of a polynomial function of degree 5, max # of turning points, with 1 real zero and a positive leading coefficient.

Rational Zero Theorem--  $\frac{p}{q}$  is a possible zero, where:

- $p \in$  set of integral factors of the constant
- $q \in$  set of the integral factors of the leading coefficient

Solve

$$0 = 6x^3 + 7x^2 - 9x + 2$$

$$\begin{aligned}
 p &\in \{\pm 1, \pm 2\} \\
 q &\in \{\pm 1, \pm 2, \pm 3, \pm 6\} \\
 \frac{p}{q} &\in \left\{ \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 2, \pm \frac{2}{3} \right\}
 \end{aligned}$$

$$\begin{array}{r}
 -2 \overline{) 6 \quad 7 \quad -9 \quad 2} \\
 \underline{-12 \quad 10 \quad -2} \\
 6 \quad -5 \quad 1 \quad 0
 \end{array}$$

$6x^2 - 5x + 1$   
 $6x^2 - 3x - 2x + 1$   
 $3x(2x-1) - 1(2x-1)$   
 $(3x-1)(2x-1)$   
 $x = \frac{1}{3} \quad x = \frac{1}{2}$   
 $\{-2, \frac{1}{3}, \frac{1}{2}\}$

Solve

$$x^4 - x^3 + 7x^2 - 9x - 18 = 0$$

$$p \in \{\pm 1 \pm 2 \pm 3 \pm 6 \pm 9 \pm 18\}$$

$$q \in \{\pm 13\}$$

$$\frac{p}{q} \in \leftarrow$$

$$\begin{array}{r}
 -1 \overline{) 1 \quad -1 \quad 7 \quad -9 \quad -18} \\
 \underline{-1 \quad 2 \quad -9 \quad 18} \\
 2 \quad 0 \quad 18
 \end{array}$$

$$\begin{array}{r}
 2 \overline{) 1 \quad -2 \quad 9 \quad -18 \quad 0} \\
 \underline{-2 \quad 0 \quad 18} \\
 1 \quad 0 \quad 9 \quad 0
 \end{array}$$

$$x^2 + 9 = 0$$

$$x^2 = -9$$

$$x = \pm 3i$$

$$\{-1, 2, \pm 3i\}$$

Do:

$$0 = 2x^4 + 3x^3 + 6x^2 + 12x - 8$$

$$\{-2, \frac{1}{2}, \pm 2i\}$$

Solve.

$$0 = x^4 - 4x^3 + 6x^2 - 8x + 8$$

$$p \in \{\pm 1 \pm 2 \pm 4 \pm 8\}$$

$$q \in \{\pm 13\}$$

$$\frac{p}{q} \in \leftarrow$$

$$\begin{array}{r}
 2 \overline{) 1 \quad -4 \quad 6 \quad -8 \quad 8} \\
 \underline{-2 \quad -4 \quad 4 \quad 8} \\
 2 \quad 0 \quad 4 \quad 0
 \end{array}$$

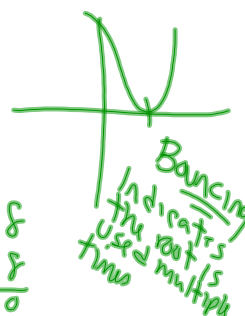
$$\begin{array}{r}
 2 \overline{) 1 \quad -2 \quad 2 \quad -4 \quad 0} \\
 \underline{-2 \quad 0 \quad 4} \\
 1 \quad 0 \quad 2 \quad 0
 \end{array}$$

$$x^2 + 2 = 0$$

$$x^2 = -2$$

$$x = \pm i\sqrt{2}$$

$$\{2, \pm i\sqrt{2}\}$$



Solve.

$$0 = x^4 - 6x^3 + 8x^2 - 48x$$

Constant = 0

Factor GCF

$$0 = x(x^3 - 6x^2 + 8x - 48)$$

$\swarrow$   
 $p \in \{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 48\}$   
 $q \in \{\pm 1\}$

$$\begin{array}{r|rrrr} 6 & 1 & -6 & 8 & -48 \\ & & 6 & 0 & 48 \\ \hline & 1 & 0 & 8 & 0 \end{array}$$

$$\{0, 6, \pm 2i\sqrt{2}\} \quad x^2 + 8 = 0$$

$$x^2 = -8$$

$$x = \pm 2i\sqrt{2}$$

HW

p381

13, 15, 19, 25, 28, 30